Application of the Modified Bartz Analysis to Very Large Nozzles

Robert L. Glick* and Reginald I. Vachon†
Thiokol Chemical Corporation, Huntsville, Ala.

The advent of large, solid propellant boosters has created a need for design information on convective heat transfer and boundary-layer growth in very large nozzles. The convective heat transfer in converging-diverging nozzles is usually computed either from empirical equations obtained from the correlation of experimental heat-transfer data or integral boundary-layer analyses. The difficulty at present is that no experimental data are available for heat transfer in very large nozzles. Therefore, the accuracy of the various methods has not been determined. However, recent results of Back, Massier, and Gier[1] show that the predictions of the modified Bartz analysis,[2] with physical properties evaluated at the film temperature, give better agreement with their experimental data than other methods, although these data are for a relatively small nozzle. Moreover, the modified Bartz analysis has been programed for automatic computation. The ability of the modified Bartz analysis to predict convective heat transfer reasonably accurately in small nozzles and the fundamental basis of this analysis recommend its use in large nozzles.

The applicability of the modified Bartz analysis to very large nozzles has not yet been established. Indeed, the experimental heat-transfer results employed to test the analysis were obtained in nozzles approximately 50 times smaller than a space-booster nozzle. The objective of this note is to report the results of a short study of the applicability of the modified Bartz analysis for predicting boundary-layer characteristics and heat transfer in a very large nozzle. In this study, the velocity boundary-layer thickness and displacement thickness predicted by the modified Bartz analysis were compared with the measured thicknesses of the boundary layer from the 16-ft supersonic wind tunnel at Arnold Engineering Development Center (AEDC).‡ Although no heat-transfer data were available for the comparison, it is believed that the comparison is still of value since convective heat transfer is intimately connected with the thickness of the boundary layer.

The nozzle in the tunnel is two-dimensional, the side walls are contoured, and the ceiling and floor are flat. The boundary-layer measurements in the tunnel were made 52.00 ft from the throat of the nozzle on the centerline of the ceiling of the tunnel. Data were recorded for exit Mach numbers of 1.5, 1.6, 1.75, and 2.0. However, since the exit Mach numbers of large booster rocket nozzles are on the order of 3.0, only the data from the Mach 2.0 test were selected for the comparison. For this test, the stagnation temperature was 750°R, and the stagnation pressure was 1000 psfa. The temperature at the tunnel wall and the temperature distribution through the thermal boundary layer were not recorded during the boundary-layer measurements at AEDC.

The modified Bartz analysis, which in general applies to axisymmetric geometry, was adapted to the two-dimensional tunnel geometry by assuming a very large, constant radius. The wall temperature was taken to be 700°R, and the free-stream pressure gradient was based on the results of one-dimensional theory. Since no initial boundary-layer thicknesses were measured, the momentum thickness at the throat, θ_i, was varied from 0.001 to 0.044 in. to determine the sensitivity of the theoretical results to the initial momentum thickness.

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* Senior Engineer. Member AIAA.

† Senior Engineer, Thiokol Chemical Corp.; now Associate Professor, Auburn University. Member AIAA.

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Table 1 Comparison of theoretical and experimental results

	Modified Bartz analysis		AEDC data
Momentum thickness at the throat, in.	0.001	0.044	...
Thickness of the velocity boundary layer, in., 52.00 ft from the throat	6.96	7.06	6.10
Thickness of the thermal boundary layer, in., 52.00 ft from the throat	5.49	6.09	no data
Displacement thickness, in., 52.00 ft from the throat	1.63	1.64	1.49

The theoretical and experimental results are presented in Table 1. The table shows that the theoretical conditions at the nozzle exit are relatively insensitive to the value selected for the momentum thickness at the nozzle throat. Table 1 shows also that the theoretical and experimental results are in reasonable agreement. There are two probable causes for the slight disagreement. First, the boundary layer on the ceiling of the tunnel is not strictly two-dimensional, since a small pressure gradient exists across the ceiling because of the curved side walls. This pressure gradient will induce a secondary flow from the center of the tunnel toward the side walls. This flow should cause a reduction in the boundary-layer thickness from that for strictly two-dimensional flow. The theoretical and experimental data show this trend. Second, the experimentally obtained velocity profiles demonstrate good agreement with a $\frac{1}{9}$ power law, rather than the $\frac{1}{7}$ power law employed in the modified Bartz analysis.

Although the comparison was limited to boundary-layer data for a single condition, the results are encouraging and give some justification for the use of the modified Bartz analysis in very large nozzles.

References

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A Method of Estimating the Effect of Shock Interaction on Stagnation Line Heating

John E. Fontenot Jr.*
The Boeing Company, New Orleans, La.

Nomenclature

a	= acoustic velocity
D	= $1 - k/[k(2-k)]^{1/2}$
d	= diameter of leading edge
E	= $\sinh^{-1} D$
k	= $2\rho_1/(\rho_2 + \rho_{2t})$
M	= Mach number
T	= absolute temperature
u	= velocity component in x direction
V	= velocity vector
v	= velocity component in y direction
x	= coordinate
y	= coordinate

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* Research Engineer, Launch Systems Branch, Aero-Space Division. Member AIAA.

$y_{\mathrm{I}},y_{\mathrm{II}}$ = limits of disturbance zone
α = sweep angle
β_1,β_2 = angles defined by Eq. (1)
γ = ratio of specific heats
δ = shock detachment distance
ρ = density
τ = shock thickness

Subscripts

1 = conditions in front of shock
2 = conditions right behind shock
L = local conditions behind shock
t = total or stagnation condition

Introduction

ON certain aerodynamic configurations, the problem of predicting stagnation line heating rates in supersonic flow is complicated by the local interaction of an extraneous shock with the bow shock. An example of such a configuration is shown in Fig. 1. From the literature[1] an estimate of the magnitude of the effect of the impinging shock on the stagnation line heating rates is available. However, no experimental data are available to permit a prediction of the width of the zone of increased heating. To provide a means of estimating the width of this zone, an approximate theory has been derived and is presented here.

Analysis

It is postulated that the disturbance created by the interaction of the extraneous shock with the bow shock can be treated as an acoustic-like disturbance. This disturbance propagates into the region behind the bow shock as a spherical wave at the local acoustic velocity. In the more general case, the leading edge is not normal to the freestream velocity but has some sweep angle, α. Consequently, there will be a component of velocity parallel to the leading edge and the disturbance will propagate further in the direction of this velocity component than in the direction opposed to it. Such a condition is depicted in Fig. 2.

From a consideration of this figure it can be seen that the maximum width of this zone is given by $y_{\mathrm{I}} + y_{\mathrm{II}}$, where

$$y_{\mathrm{I}} = \int_{\delta}^{0} \tan\beta_1 dx \qquad y_{\mathrm{II}} = \int_{\delta}^{0} \tan\beta_2 dx \tag{1}$$

A further consideration of this figure gives

$$\tan\beta_1 = (a_2 + v)/u_L \qquad \tan\beta_2 = (a_2 - v)/u_L \tag{2}$$

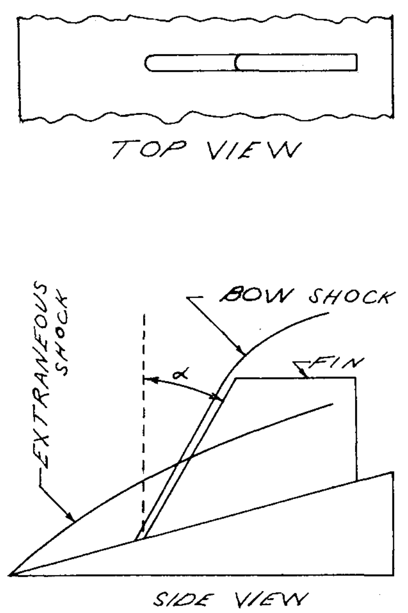

Fig. 1 Wedge-swept fin configuration.

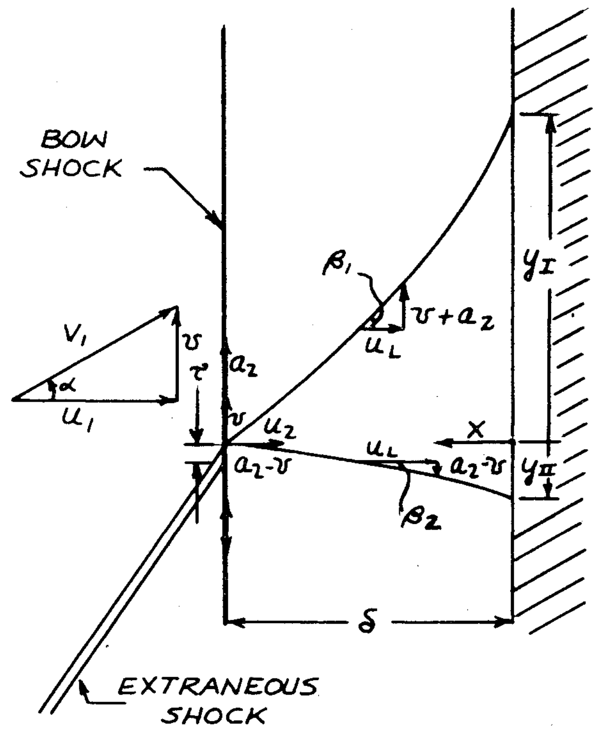

Fig. 2. Flow model.

For hypersonic flow, Truitt[2] gives an expression for u_L as

$$\frac{u_L}{u_1} = \frac{k[k(2-k)]^{1/2}}{1-k} \sinh\left\{\frac{x}{\delta}\sinh^{-1}\frac{(1-k)}{[k(2-k)]^{1/2}}\right\} \tag{3}$$

or

$$u_L/u_1 = (k/D)\sinh[(E/\delta)x] \tag{4}$$

Now

$$dy_{\mathrm{I}} = \tan\beta_1 dx = [(a_2 + v)/u_L]dx \tag{5}$$

or

$$y_{\mathrm{I}} = -\frac{(a_2+v)D}{k\,u_1}\int_0^{\delta}\frac{dx}{\sinh[(E/\delta)x]} \tag{6}$$

Thus

$$\frac{y_{\mathrm{I}}}{\delta} = -\frac{D}{kE}\left[\ln\left(\tanh\frac{E}{2}\right)\right]\frac{a_2+v}{u_1} \tag{7}$$

Similarly

$$\frac{y_{\mathrm{II}}}{\delta} = -\frac{D}{kE}\left[\ln\left(\tanh\frac{E}{2}\right)\right]\frac{a_2-v}{u_1} \tag{8}$$

or

$$\frac{y_{\mathrm{I}}}{\delta} = -\frac{D}{kE}\left[\ln\left(\tanh\frac{E}{2}\right)\right]\left[\frac{(T_2/T_1)^{0.5} + M_1\sin\alpha}{M_1\cos\alpha}\right] \tag{9a}$$

and

$$\frac{y_{\mathrm{II}}}{\delta} = -\frac{D}{kE}\left[\ln\left(\tanh\frac{E}{2}\right)\right]\left[\frac{(T_2/T_1)^{0.5} - M_1\sin\alpha}{M_1\cos\alpha}\right] \tag{9b}$$

In the derivation of Eq. (3) it is assumed that the value of k is a constant and that δ/d is small.[2] At hypersonic Mach numbers, these approximations are not bad, but their validity at supersonic Mach numbers is questionable. Despite this fact, calculations discussed below were carried out for $M_1 \cos\alpha$ between 2 and 7. In writing Eq. (9), it is further assumed that the sound speed in the region behind the shock is

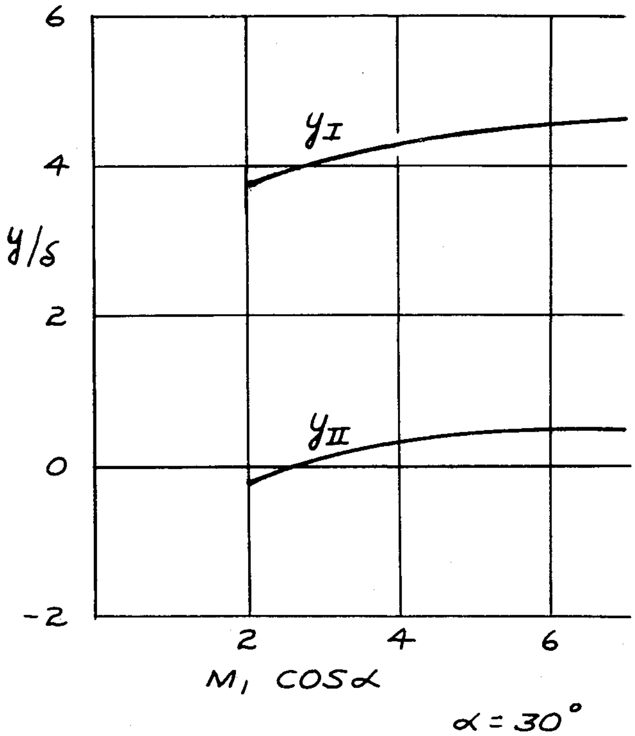

Fig. 3. y/δ vs normal component of M_1.

constant and equal to that immediately behind the shock. The error introduced by this assumption is quite small.

Now $k = 2\rho_1/(\rho_2 + \rho_{2t})$ is a function only of $M_1 \cos\alpha$, and D and E are similarly dependent. Also from Ref. 3

$$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 \cos^2\alpha - (\gamma - 1)][(\gamma - 1)M_1^2 \cos^2\alpha + 2]}{(\gamma + 1)^2 M_1^2 \cos^2\alpha} \tag{10}$$

It is thus seen that y_I/δ and y_{II}/δ are functions only of M_1 and α. These two nondimensional parameters are plotted in Fig. 3 as a function of M_1 for $\alpha = 30°$.

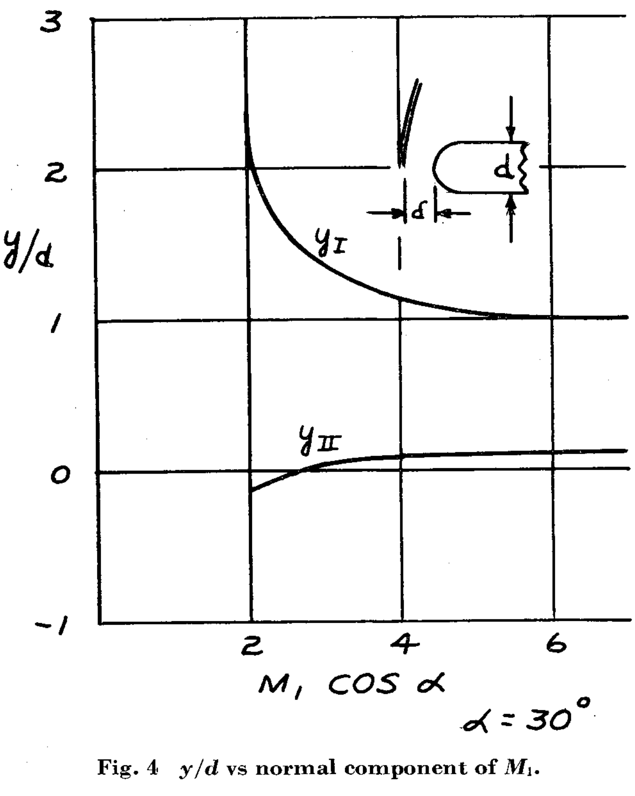

Fig. 4 y/d vs normal component of M_1.

Liepmann and Roshko[4] present a compilation of experimental data for δ/d as a function of Mach number for a cylindrical leading edge. This data was used and values of y_I/d and y_{II}/d were determined, and are shown as a function of M_1 for $\alpha = 30°$ in Fig. 4. From this figure it can be seen that the maximum width of the zone of increased heating is approximately constant for normal Mach numbers greater than 5, and has a magnitude of less than one leading edge diameter. It is noted that at very high altitudes, where the mean free path is large, the value of τ, the impinging shock thickness, becomes substantial and should be considered in estimating the width of the interaction zone.

Conclusions

Combining the experimental data of Ref. 1 with the foregoing analysis, a method to predict the effect of shock interaction on leading edge heating is available. Lacking experimental verification, this method is only a promising approach.

References

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Simulation of the Interaction of a Hypersonic Body and a Blast Wave

Gene J. Bingham* and Theodore E. Davidson†
North American Aviation, Inc., Columbus, Ohio

A TRANSIENT pressure rise will be produced at the surface of a hypersonic body intersected by a blast wave. The transient pressure will be a function of the strength of the body wave and the blast wave plus the attitude of the interacting waves. The general characteristics of body waves and blast waves have been defined individually. However, the interaction effects of the two wave systems have not been completely defined. The investigation described here was initiated to establish the feasibility of a technique to simulate and to measure the interaction of a blast wave and a hypersonic body.

A model mounted in a continuous hypersonic wind tunnel simulated the hypersonic body. A shock tube was selected to simulate the blast wave because 1) the strength of the traveling shock wave is controllable and repeatable, 2) safety problems are minimized, 3) a shock tube can be added to open jet tunnels, 4) the physical parameters associated with shock tubes are well known, and 5) feasibility of this technique had been demonstrated with a supersonic wind tunnel with a solid jet boundary.[1]

It is noted that, although a supersonic tunnel-shock tube arrangement permitted an investigation of shock interaction, the feasibility with a hypersonic wind tunnel had not been established. In particular, the question remained as to

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* Senior Technical Specialist, Advanced Environmental Development Group. Member AIAA.

† Senior Engineer, Advanced Environmental Development Group.